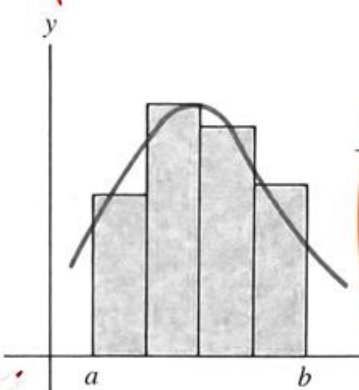
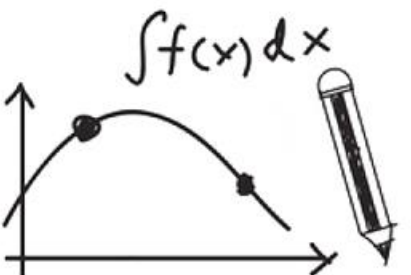




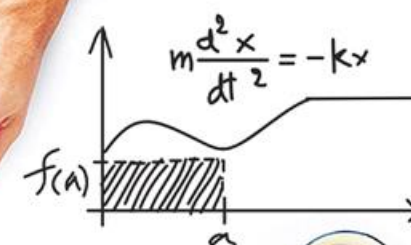
$$x^2 - 3x - 4 = 0$$
$$4x^2 - 3x - 1 = 0$$



# Calculus(I)

$$\frac{d}{dx} \left[ \frac{f(x)}{g(x)} \right] = \frac{g(x)f'(x) - f(x)g'(x)}{g(x)^2}$$

$$F = mg = ma = m \frac{d^2h}{dt^2}$$



Gottfried Wilhelm Leibniz

$$\frac{dA}{dt} = \frac{dB}{dt} = -\frac{dC}{dt} = \frac{dD}{dt} = (c_1)T^{\frac{1}{2}}AB - (c_2)T^{\frac{1}{2}}CD$$

$$m \frac{d^2x}{dt^2} = -kx - f \frac{dx}{dt} + A \sin(\omega t)$$
$$y' = \text{and } v' = -ky - fv + A \sin(\omega t)$$

$$m \frac{d^2x}{dt^2} = -kx$$

$$x = A \cos(\omega t + \phi)$$
$$\frac{dA}{dt} = (c_1)(T - T)$$

$$\frac{df(x)}{dx}$$

$$\frac{b^2 - 4ac}{4a^2}$$

$$x + \frac{b}{2a} = \frac{\sqrt{b^2 - 4ac}}{2a}$$

$$x + \frac{b}{2a} = -\frac{\sqrt{b^2 - 4ac}}{2a}$$

$$x + h, f(x + \tau)$$



# Rigorous Study of Limits

Lecturer: Xue Deng

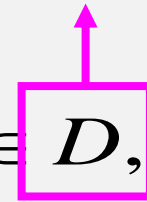
# Preliminaries

domain



**Definition:**

$$f : D \rightarrow E, \quad x \mapsto y = f(x), \quad x \in D,$$



**Range:**

$$R_f = f(D) = \{y \mid y = f(x), x \in D\}.$$

# Note

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(1)

**Two elements:**  $D_f$  and the function  $f$ .

**Eg :**  $y = \lg x^2$ ,  $y = 2\lg x$  are two different functions.

(2)


$y = g(x)$ ,  $y = F(x)$ ,  $y = \phi(x)$  Etc ,  $\mathbf{y = y(x)}$

(3)

$f(x) = f(t) = f(u) = \dots$

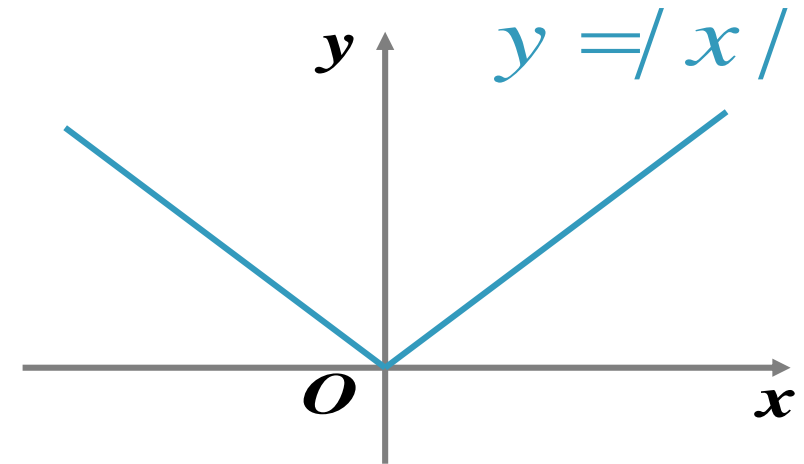
# Preliminaries: Some important functions

## Eg 1: Absolute function

  $y = |x| = \begin{cases} x, & x \geq 0, \\ -x, & x < 0 \end{cases}$

 Domain:  $D = (-\infty, +\infty),$

Range:  $R_f = [0, +\infty).$

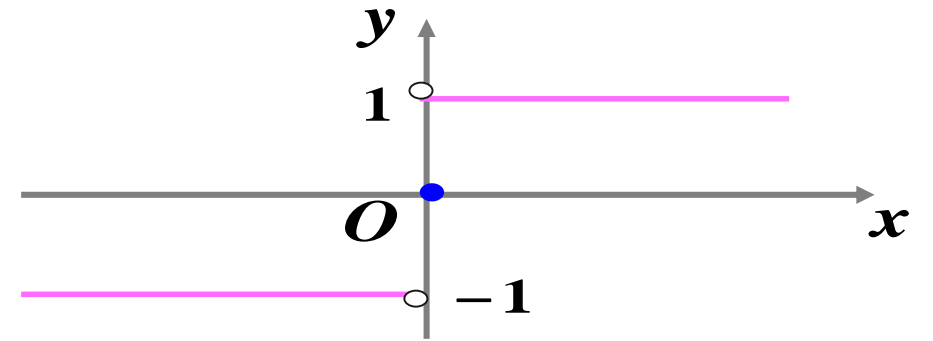


# Preliminaries

## Eg 2: Sign function



$$y = \operatorname{sgn} x = \begin{cases} 1, & x > 0, \\ 0, & x = 0, \\ -1, & x < 0 \end{cases}$$



Domain:  $D = (-\infty, +\infty)$ ,

Range:  $R_f = \{-1, 0, 1\}$ .

As to  $\forall x \in \mathbf{R}$ , have  $x = \operatorname{sgn} x \cdot |x|$  or  $|x| = x \cdot \operatorname{sgn} x$ .

# Preliminaries

Eg 3: Integer function

Maximum integer which is not more than  $x$



$$y = [x] = n, \quad \text{when } n \leq x < n + 1, n \in \mathbb{Z}$$

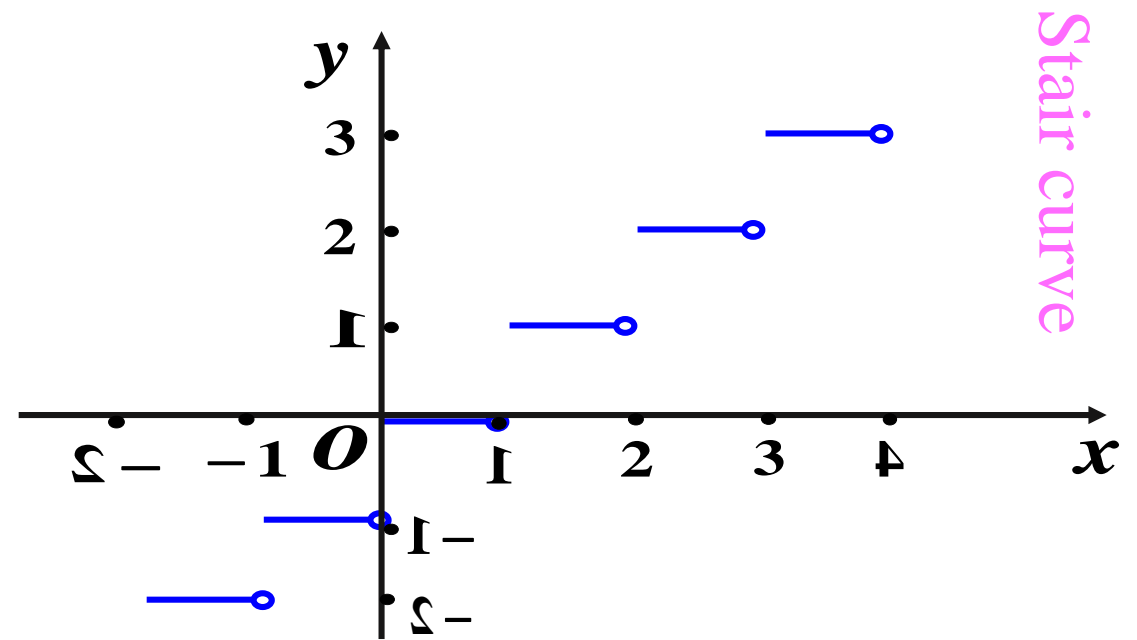
Like:  $[2.5] = 2$

$$[5.2] = 5$$

$$[7.9] = 7$$

$$[5] = 5$$

$$[-2.5] = -3$$



Domain:  $D = (-\infty, +\infty)$ ,

Range: integers

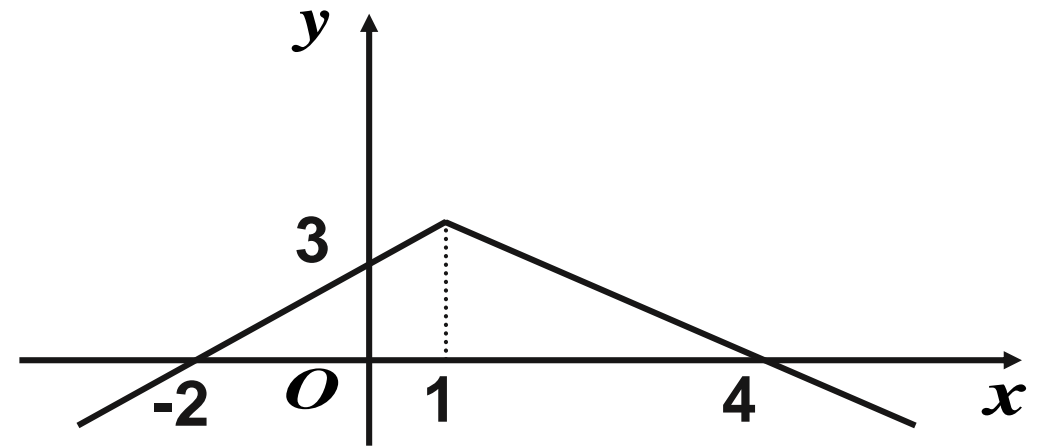
# Preliminaries

Eg 4: Express the function  $y = 3 - |x - 1|$  by using piece-wise function.



$$y = \begin{cases} 3 + (x - 1), & x < 1 \\ 3 - (x - 1), & x \geq 1 \end{cases}$$

$$\text{Namely, } y = \begin{cases} 2 + x, & x < 1 \\ 4 - x, & x \geq 1 \end{cases}$$





# Rigorous Study of Limits

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